Determination of the heat transfer coefficient h in a cooling tube

A.Hormière, ST-CV, 17/08/2001

Abstract

This document gives the details of the determination of the heat transfert coefficient in a cooling tube. It allows the calculation of the temperature difference between the mean water temperature T_m and the inside surface temperature of the pipe T_s.

Introduction

When the water flows in a cooling tube, a thermal boundary layer is developing. There is a temperature gradient between the internal surface of the pipe and the water to allow heat transfer. There is also a gradient inside the water (Figure 1).
It is very difficult to know precisely the temperature gradient inside the water, for this reason, we use the mean temperature value T_m, which is the averaged value of the temperature at a position x.
The heat transfer coefficient h between T_m and the inside surface temperature T_s can be evaluated using correlations on the Nusselt Number (Nu) which represent the ration between convection and conduction heat transfer.
As we know h, we can evaluate the temperature difference between T_m and T_s for a certain heat flux ϕ.

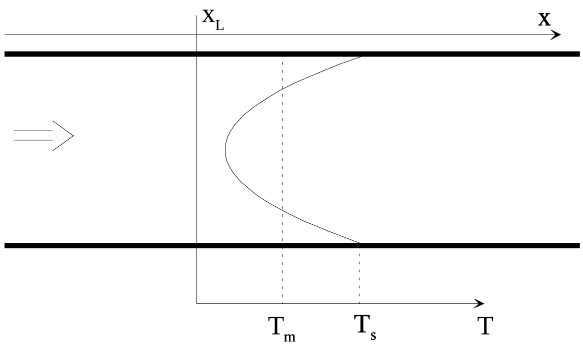

Figure 1: Temperature gradient in a cooling pipe $T_s > T_m$, at $x = x_L$

Correlations

Lets consider a tube of internal diameter D, with a fluid flow of speed U. The fluid is characterized by its density ρ, thermal capacity C_p, thermal conductivity λ and dynamic viscosity μ.
The Nusselt number is adimensional and defined as: $Nu = \frac{h\,D}{\lambda}$. As we know Nu, we can determine h.

In this type of flow, Nu is a constant for laminar flow and a fonction of two other adimensional numbers, Pr and Re, $Nu = Nu(Re, Pr)$ for turbulent flows. Pr depends only of the fluid used, it is defined as $Pr = \frac{\nu}{a}$ wher ν is the kinematic viscosity $\nu = \frac{\mu}{\rho}$ and a the thermal diffusivity $a = \frac{\lambda}{\rho\,C_p}$. For the water $Pr \simeq 7.5$. Re depends on the fluid an the motion conditions, $Re = \frac{U\,d}{\nu}$.

We use the *Dittus-Boelter* equation from [1] for turbulent flow ($Re > 10^4$) and the value 4.36 for laminar flow ($Re < 2500$). Between this range, we assume a linear behavior to cover the whole range of flows. The equation is valid after a certain length of pipe, typically $10D$.
The following equation is only valid for water, as we need to fix a Pr to make the linear approximation. It can easily be extended to another type of fluid.

$$Nu = \begin{cases} 4.36 & \text{if } Re < 2500 \\ \frac{81.72-4.36}{7500}\, Re - 21.46 & \text{if } 2500 < Re < 10^4 \\ 0.023\, Re^{\frac{4}{5}}\, Pr^{0.4} & \text{if } Re > 10^4 \text{ and } 0.7 < Pr < 160 \end{cases}$$

When we have the Nu, we apply: $h = \frac{Nu\,\lambda}{D}$.

In the Figure 2 we can see that Nu increases with Re, which means for a pipe of a diameter D, Nu ie h increases with the speed U.

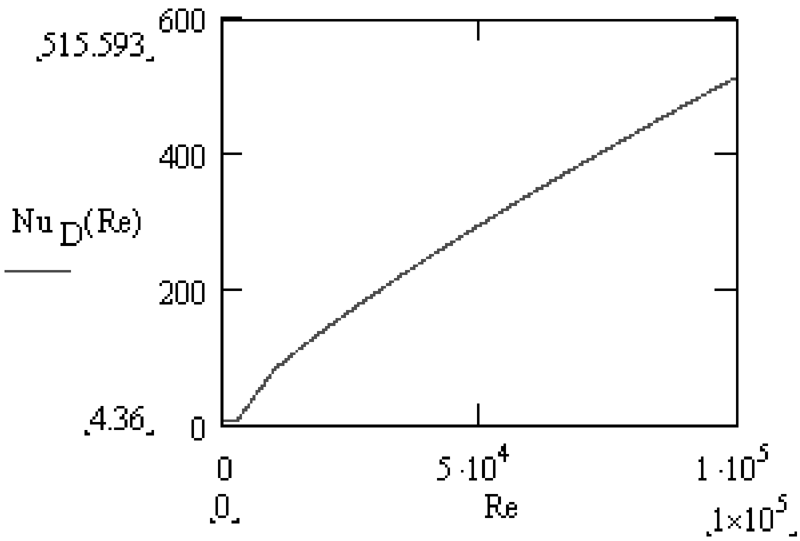

Figure 2: Nusselt number fonction of Reynolds for a water cooling pipe

References

[1] Franck P. Incropera and David P. DeWitt. *Fundamentals of Heat and Mass Transfer.* John Wiley & Sons, 1996.