Heat transfert calculation in a tube

Abstract

This document gives the details of a heat transfert calculation in an insulated tube to find the temperature rising.

Let's consider a pipe like in the figure 1. We apply the conservation of energy on the volume. The heat that enters the volume is $\dot{H}_1$ with $\dot{H}_1 = \dot{m} C_p T_1$ where $\dot{m}$ is the mass flow rate and C_p the heat capacity. ϕ is the heat flux on the tube from the outside, by length of the tube (W/m).

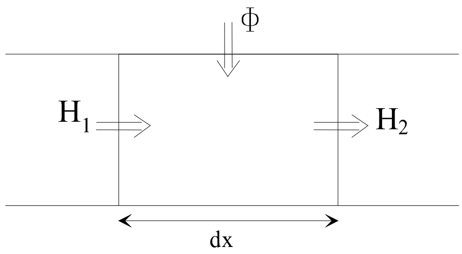

Figure 1: Heat exchange on a pipe chunk

The conservation of the energy gives us:

$$\dot{H}_2 = \dot{H}_1 + \phi\, dx$$

That gives:

$$\frac{\partial \dot{H}}{\partial x} = \phi \tag{1}$$

We have now to define the heat flux ϕ per meter of tube. We only have to make a bi-dimensional analyse on a section of the tube.

The parameters are T_{in} the temperature of the fluid in this section. We suppose that the wall of the tube is at the same temperature (thickness neglected). T_{ext} the temperature of the ambient. r_1 and r_2, respectively the external radius of the tube and the radius of the tube with insulation. λ is the heat conduction coefficient of the insulation and h the global heat exchange coefficient with the ambient (convection + radiations).

We can write:

$$\phi = \frac{T_{ext} - T_{in}}{R} \tag{2}$$

With R the global resistance of the air and the insulation.

$$R = \frac{\ln(r_2/r_1)}{2\pi\lambda} + \frac{1}{h\, 2\pi r_2}$$

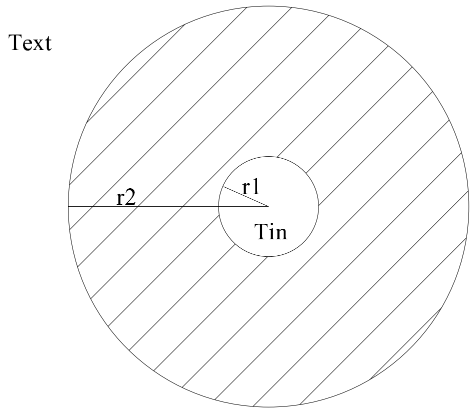

Figure 2: Section of an insulated pipe

From (1) , (2) and the definition of $\dot{H}$, we can write:

$$\frac{\partial T}{\partial x} = \frac{-\left(T(x) - T_{ext}\right)}{\dot{m} C_p \, R}$$

$$\Longleftrightarrow \frac{\partial \theta}{\partial x} = \frac{-\theta(x)}{\dot{m} C_p \, R}$$

That, after some calculations gives:

$$\boxed{T(x) = \left(T_{x=0} - T_{ext}\right) \exp^{-\frac{x}{\dot{m} C_p \, R}} + T_{ext}}$$